Modern Physics Letters A

A NEW DESCRIPTION OF COSMIC STRINGS IN BRANE WORLD SCENARIO

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In the light of ϕ-mapping topological current theory, the structure of cosmic strings are obtained from the Abelian Higgs model, which is an effective description to the brane world cosmic string system. In this topological description of the cosmic string, combining the result of decomposition of U(1) gauge potential, we analytically reach the familiar conclusions that in the brane world scenario the magnetic flux of the cosmic string is quantized and the RR charge of it is screened.

Keywords: Brane world; cosmic strings.

PACS Nos.: 11.25.Uv, 11.27.+d, 02.40.-k

The current WMAP data has placed the inflationary cosmology on solid grounds.[1,2,3,4] So the brane inflation[5] has become an indispensable part of the brane world scenario. In a particularly simple version,[6,7,8,9,10] brane inflation takes place when a D3-brane and an anti-D3-brane move slowly towards each other, and ends as the D3-brane pair annihilates. Suppose a stack of D3-branes spans our universe, and the Standard Model fields are open modes on them. When one of them is anihilated at the end of the inflation, the D-strings as the cosmic strings will be produced inside (or very close to) the D3-branes. Assuming all extra dimensions are compactified, the stability of such D-strings as D1-vortices inside the D3-branes was demonstrated.[11] It is also pointed out in Ref. 11 that the simplest way to see the existence and stability of D1-vortices in D3-branes is to realize the direct connection between the D1-vortex solution and a certain limit of Abelian Higgs (AH) model. So the study of the AH model is very important to the description of the cosmic strings in the brane world scenario.

To construct the AH model, the complex scalar Higgs field ϕ and U(1) gauge field A_μ inside the D3-brane are introduced. The phase of the Higgs field ϕ is denoted by the axion φ , which is the dual of the RR 2-form field $C_{\mu\nu}$ inside the D3-brane. The axion φ measures the winding number of the D1-vortex and the field $C_{\mu\nu}$ measures

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the net RR charge of the D1-vortex. Then the AH model lagrangian is

$$L = -(D^\mu \phi)^* D_\mu \phi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{\lambda}{4}(\phi^* \phi - v^2)^2 \quad (\mu, \nu = 0, 1, 2, 3), \tag{1}$$

where $D_\mu = \partial_\mu + ieA_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

It is well known that the AH model has vortex solution (Abrikosov-Nielsen-Olesen vortex [12,13]). Using so-called ϕ-mapping topological current theory, this vortex solution can be expressed in another way.[14] And this kind of expression provides a new description of the cosmic string. In fact, according to the ϕ-mapping topological current theory, the vortex (or string) configurations can always be produced from the zero points of the complex scalar field in the four-dimensional spacetime. For example, the cosmic string structures originated from the zero points of the complex scalar quintessence field were discussed in Ref. 15. The advantage of this description of the vortex configurations is the physical quantities concerning the topology of the system can be expressed in an analytical way, and the relationships of these quantities can be proved strictly. Therefore, for the purpose of studying the topological property of the cosmic strings, this description is highly significant.

In the present work, using the ϕ-mapping topological current theory, we obtain the cosmic string structures from the AH model, which is an effective description to the brane world cosmic string system. Furthermore, combining the result of decomposition of U(1) gauge potential in Ref. 19, we analytically reach the familiar conclusions that in the brane world scenario the magnetic flux of the cosmic string is quantized and the RR charge of it is screened.[11]

To be self-contained basically, we will briefly review the ϕ-mapping topological current theory in the following. Consider a D-dimensional smooth manifold with metric tensor $g_{\mu\nu}$ and local coordinates x^μ. Define a map Φ:

$$\Phi^a = \Phi^a(x^\mu) \quad (a = 1, \cdots, d < D), \tag{2}$$

and introduce the direction unit field of Φ^a:

$$N^a = \frac{\Phi^a}{\|\Phi\|} \quad (\|\Phi\| = \sqrt{\Phi^a \Phi^a}). \tag{3}$$

Then we can construct a topological tensor current:

$$j^{\mu\cdots\nu} \sim \frac{1}{\sqrt{detg_{\mu\nu}}}\, \epsilon^{\mu\cdots\nu\lambda\cdots\rho} \epsilon^{a\cdots b} \partial_\lambda N^a \cdots \partial_\rho N^b. \tag{4}$$

It is easy to see that $j^{\mu\cdots\nu}$ is completely antisymmetric and identically conserved. Define the Jacobian tensor $D^{\mu\cdots\nu}(\frac{\Phi}{x}) = \frac{1}{d!}\, \epsilon^{\mu\cdots\nu\lambda\cdots\rho}\epsilon^{a\cdots b}\partial_\lambda \Phi^a \cdots \partial_\rho \Phi^b$, and the ϕ-space Green function $G_d(\|\Phi\|) = \begin{cases} \frac{1}{\|\Phi\|^{d-2}}, & d > 2; \\ \ln\|\Phi\|, & d = 2. \end{cases}$ Then using $\partial_\mu N^a = \frac{1}{\|\Phi\|}\partial_\mu \Phi^a + \Phi^a \partial_\mu \frac{1}{\|\Phi\|}$ and the Green function relation $\frac{\partial}{\partial \Phi^a}\frac{\partial}{\partial \Phi^a} G_d(\|\Phi\|) \sim \delta^d(\Phi)$ we can find

$$j^{\mu\cdots\nu} \sim \frac{1}{\sqrt{detg_{\mu\nu}}}\, \delta^d(\Phi) D^{\mu\cdots\nu}(\frac{\Phi}{x}), \tag{5}$$

which implies that $j^{\mu\cdots\nu} \neq 0$ only at the points where $\Phi^a = 0$. Generally, these points would correspond to the submanifolds where the topological defects are located. If we denote the k-th above-mentioned submanifold by M_k, and define a corresponding normal submanifold N_k which is spanned by the parameter v^A $(A = 1, \cdots, k)$ with the metric tensor g_{AB}, we can get the only intersection point of M_k and N_k denoted by p_k. By virtue of the implicit function theorem,[17] at the regular points of Φ^a, there exists the only solution to the equations $\Phi^a = 0$, and we can expand Eq. (5) as

$$j^{\mu\cdots\nu} \sim \frac{1}{\sqrt{detg_{\mu\nu}}} \sum_k \frac{W_k \sqrt{detg_{AB}}}{D(\frac{\Phi}{v})\big|_{p_k}} \delta^d(M_k) D^{\mu\cdots\nu}(\frac{\Phi}{x}), \tag{6}$$

where the Jacobian $D(\frac{\Phi}{v}) = \frac{1}{d!} \epsilon^{A\cdots B} \epsilon^{a\cdots b} \partial_A \Phi^a \cdots \partial_B \Phi^b$, $\delta^d(M_k)$ is the δ-function on the submanifold M_k, and W_k denotes the winding number of the k-th topological defect. Some important physical quantities concerning the topology, say magnetic flux, can be calculated from Eq. (6). According to the implicit function theorem,[17] the irregular points of Φ^a correspond to the branch points of the topological current $j^{\mu\cdots\nu}$, and the branch processes of $j^{\mu\cdots\nu}$ occur at these very points. These branch processes can describe various evolutions of the topological defects, and the total topological charge of the system will keep unchange during these evolutions since $j^{\mu\cdots\nu}$ is a conserved current.

Before entering into the concrete analysis of the AH model (1), there remains two more issues that are worth noting. First, our description of the cosmic string is different from the exact physical picture of it in a subtle way, since the starting action (1) is different from the action used in Ref. 11 where a δ-function source term for the RR field is introduced. When we discuss the topological property of the cosmic string, this difference can be ignored. Second, as in Ref. 11, our study to the cosmic string structure originate from the AH model (1), but the conclusions we obtain are not limited to it. For example, the RR charge of the cosmic string which we will discuss below is absent from the original AH model. Actually, our discussions apply to the general situation provided by Ref. 11.

When the spontaneous symmetry breaking takes place in the system described by the AH model lagrangian (1), the gauge field A_μ swallows the axion φ , which serves as the Goldstone Boson, and gains mass

$$m_A = \sqrt{2} e v. \tag{7}$$

Besides, the Higgs boson also gains mass

$$m_H = \sqrt{\lambda} v. \tag{8}$$

The value of the ratio $\beta = m_H^2/m_A^2$ will decide the force between the vortices,[16] so it is an important parameter in the study of the branch processes of the cosmic strings. Define

$$\begin{aligned} J_\mu &= -ie[\phi^*(D_\mu\phi) - (D_\mu\phi)^*\phi] \\ &= 2e^2 A_\mu \phi^*\phi - ie[\phi^*(\partial_\mu\phi) - (\partial_\mu\phi)^*\phi]. \end{aligned} \tag{9}$$

Then, from (1), we have the equations of motion

$$D^\mu D_\mu \phi - \frac{\lambda}{2}(\phi^* \phi - v^2)\phi = 0, \tag{10}$$

$$\partial_\mu F^{\mu\nu} = J^\nu. \tag{11}$$

Set $\phi = \phi^1 + i\phi^2$, $\phi^*\phi = \|\phi\|^2 = \phi^a\phi^a (a = 1, 2)$. We can find

$$J_\mu = 2e^2 A_\mu \|\phi\|^2 + 2e(\phi^1 \partial_\mu \phi^2 - \phi^2 \partial_\mu \phi^1), \tag{12}$$

and separate (10) into two real equations

$$\Box\phi^1 - e\partial^\mu A_\mu \phi^2 - 2eA^\mu \partial_\mu \phi^2 - e^2 A^2 \phi^1 - \frac{\lambda}{2}(\|\phi\|^2 - v^2)\phi^1 = 0, \tag{13}$$

$$\Box\phi^2 + e\partial^\mu A_\mu \phi^1 + 2eA^\mu \partial_\mu \phi^1 - e^2 A^2 \phi^2 - \frac{\lambda}{2}(\|\phi\|^2 - v^2)\phi^2 = 0, \tag{14}$$

where $\Box = \partial^\mu \partial_\mu$ and $A^2 = A^\mu A_\mu$. From above two equations, the current J^μ is found to be conserved:

$$\partial_\mu J^\mu = 0. \tag{15}$$

This result is consistent with Eq. (11).

Using the ϕ-mapping topological current theory, we can develop the cosmic strings structures from the Higgs field ϕ. Introduce the two-dimensional unit vector n^a from ϕ^a:

$$n^a = \frac{\phi^a}{\|\phi\|}. \quad (a = 1, 2) \tag{16}$$

Define the topological tensor current

$$j^{\mu\nu} = -\frac{1}{e}\epsilon^{\mu\nu\lambda\rho}\epsilon_{ab}\partial_\lambda n^a \partial_\rho n^b. \tag{17}$$

Obviously, $j^{\mu\nu}$ is anti-symmetric and identically conserved. Similar to Ref. 15, we can obtain

$$j^{\mu\nu} = -\frac{2\pi}{e}\delta^2(\phi) D^{\mu\nu}(\frac{\phi}{x}), \tag{18}$$

where $\delta^2(\phi)$ is the ϕ-space δ-function, which satisfies the Green function relation $\frac{\partial}{\partial\phi^a}\frac{\partial}{\partial\phi^a} ln\|\phi\| = 2\pi\delta^2(\phi)$, and $D^{\mu\nu}(\frac{\phi}{x}) = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}\epsilon_{ab}\partial_\lambda\phi^a\partial_\rho\phi^b$ is the Jacobian tensor. Assume the normal submanifolds of the world sheets of the cosmic strings are spanned by the parameter v^1 and v^2. Then from the implicit function theorem,[17] it follows that under the regular condition

$$D(\frac{\phi}{v}) \equiv \frac{1}{2}\ \epsilon^{ab}(\frac{\partial\phi^a}{\partial v^1}\frac{\partial\phi^b}{\partial v^2} - \frac{\partial\phi^a}{\partial v^2}\frac{\partial\phi^b}{\partial v^1}) \neq 0, \tag{19}$$

there exists the only solution to $j^{\mu\nu} \neq 0$:

$$x^\mu = x^\mu_k(u^1, u^2) \quad (k = 1, \cdots, l), \tag{20}$$

which represents l two-dimensional world sheets of the cosmic string with intrinsic coordinates u^1 and u^2. Though we obtain above cosmic string structures from the

Higgs field ϕ, it is not hard to see that the same result can also be obtained from the more general complex scalar field with its vacuum manifold S^1. Furthermore, the present of the gauge field A_μ will provide the possibility of the finite energy for the unit length of the cosmic string.

Using δ-function theory,[18] we can expand (18) in a way with more specific topological meaning[15]

$$j^{\mu\nu} = -\frac{2\pi}{e} D^{\mu\nu}(\frac{\phi}{x}) \sum_{k=1}^{l} \frac{W_k}{D(\frac{\phi}{v})\Big|_{p_k}} \int_{S_k} \delta^4(x^\mu - x_k^\mu(u))\sqrt{g_u} d^2u, \tag{21}$$

where W_k is the winding number of the k-th cosmic string, the Jacobian $D(\frac{\phi}{v})$ is defined in Eq. (19), p_k is the intersection point of the k-th worldsheet and its normal submanifold, $\delta^4(x^\mu - x_k^\mu(u))$ denotes the δ-function in the four-dimensional space-time, S_k represents the world sheet of the k-th cosmic string, and $\sqrt{g_u} d^2u$ is the invariant volume element of the world sheet. In light of the implicit function theorem,[17] the spatial components of $j^{\mu\nu}$ is

$$\begin{aligned} j^i = j^{0i} &= -\frac{2\pi}{e} D^i(\frac{\phi}{x}) \sum_{k=1}^{l} \frac{W_k}{D(\frac{\phi}{v})\Big|_{p_k}} \int_{S_k} \delta^4(x^\mu - x_k^\mu(u))\sqrt{g_u} d^2u \\ &= -\frac{2\pi}{e} D^i(\frac{\phi}{x}) \sum_{k=1}^{l} \frac{W_k}{D(\frac{\phi}{v})\Big|_{p_k}} \int_{L_k} \delta^3(\vec{x} - \vec{x}(s)) ds \\ &= -\frac{2\pi}{e} \sum_{k=1}^{l} W_k \frac{dx^i}{ds} \int_{L_k} \delta^3(\vec{x} - \vec{x}(s)) ds, \end{aligned} \tag{22}$$

where $D^i(\phi/x) = \frac{1}{2}\epsilon^{ijk}\epsilon_{ab}\partial_j\phi^a\partial_k\phi^b \quad (i,j,k = 1,2,3)$ is the Jacobian vector, L_k represents the k-th cosmic string, dx^i is the line element of the i-direction, and ds is the line element along the cosmic string direction.

From (9), we have

$$\begin{aligned} A_\mu &= \frac{i}{2e\phi^*\phi}(\phi^*\partial_\nu\phi - \partial_\nu\phi^*\phi) - \frac{i}{2e\phi^*\phi}[\phi^*(D_\nu\phi) - (D_\nu\phi)^*\phi] \\ &= -\frac{1}{e}\epsilon_{ab}n^a\partial_\mu n^b + \frac{1}{e}\epsilon_{ab}n^a D_\mu n^b. \end{aligned} \tag{23}$$

According to the result of decomposition of the U(1) gauge potential,[19] Eq. (23) can be expressed as

$$A_\mu = -\frac{1}{e}\epsilon_{ab}n^a\partial_\mu n^b - \frac{1}{e}\partial_\mu\Theta, \tag{24}$$

where Θ is a phase factor. So the corresponding field tensor is

$$F_{\mu\nu} = -\frac{2}{e}\epsilon_{ab}\partial_\mu n^a\partial_\nu n^b. \tag{25}$$

The magnetic field

$$B^i = \frac{1}{2}\epsilon^{ijk}F_{jk} = -\frac{1}{e}\epsilon^{ijk}\epsilon_{ab}\partial_j n^a \partial_k n^b = j^i, \tag{26}$$

i.e. the mathematical form of the magnetic field is identical to that of the spatial component of $j^{\mu\nu}$ defined in (22). Therefore, using (22) and (26), we can find the magnetic flux of the kth cosmic string is

$$\Phi_k = \int_{\Sigma_k} B^i d\sigma_i = -\frac{2\pi}{e}W_k, \tag{27}$$

where Σ_k is the two-dimensional spatial surface perpendicular to L_k. As showed in (27), once more, we obtain the familiar result that the magnetic flux of the Abrikosov-Nielsen-Olesen vortex is quantized. The derivation which has the similar mathematical form can also be found in Ref. 19. It is worth noting that, during the course of this derivation, there is no need to assume the current J^μ vanishes and the inner structure of A_μ and $F_{\mu\nu}$ (or B^i) are evident.

From Eqs. (9), (23) and (24), it is easy to see that

$$J_\mu = -2e\|\phi\|^2\partial_\mu\Theta. \tag{28}$$

In terms of the RR 2-form field $C_{\mu\nu}$, we have[11]

$$\epsilon^{\mu\nu\lambda\rho}\partial_\nu C_{\lambda\rho} = \frac{1}{m_A}J^\mu = -\frac{2e}{m_A}\|\phi\|^2\partial_\mu\Theta. \tag{29}$$

For the cylindrical symmetry around a certain cosmic string, $\|\phi\|^2$ is only the function of the distance from the cosmic string. So

$$\oint \epsilon^{\mu\nu\lambda\rho}\partial_\nu C_{\lambda\rho} d\theta = 0. \tag{30}$$

Therefore, from Eqs. (27) and (30), we can also draw the conclusion, which has been given in Ref. 11, that the winding number contribution from ϕ cancels the magnetic flux contribution from A_μ so that the net RR charge of the cosmic string is zero. From the derivation of Eq. (30), we can see that the above conclusion is a topological property of the cosmic string, and independent of the detail of the system. So, when two cosmic strings get close enough to each other, and the cylindrical symmetry of the field distribution around a single cosmic string is broken, we could expect that the Eq. (30) would not hold for a single cosmic string any more, but the total RR charge of the cosmic strings would still be zero globally for the topological reason.

In conclusion, by virtue of ϕ-mapping topological current theory, the structure of cosmic strings has been obtained from the AH model, which provides an effective description to the brane world cosmic string system. As mentioned before, this description of the cosmic string is very similar to that of the vortex in the type-II superconductor, the fact that the cosmic strings is originated from the zero points of the Higgs field ϕ is manifested obviously. This is different from the exact physical picture of the cosmic string mentioned in Ref. 11, that the cosmic strings should be thought as the points where the phase of the Higgs field ϕ is not defined, even with

$\|\phi\| = 1$ there. In above-mentioned description, combining the result of decomposition of U(1) gauge potential, we have reached the familiar conclusions again as follows: i) the magnetic flux of the cosmic string is quantized, ii) the RR charge of the cosmic string is screened.

It is easy to see that the approach used in this paper expresses the inner structures of the physical quantities analytically and has the highly topological meaning. Using this approach it has been proved that when the regular condition (19) is fail, the branch processes of the cosmic strings will occur, and in these processes the total winding number as the topological charge of the cosmic strings is conserved.[20] We can expect that the generalized ϕ-mapping topological current theory[21,22,23] and the decomposition theory of the gauge potential beyond U(1)[24,25,26,27] will play a more important role in the future study of the cosmic strings and other topological defects.

Acknowledgements

This work was supported by the National Natural Science Foundation of the People's Republic of China (No. 502-041016) and the Fundamental Research Fund for Physics and Mathematics of Lanzhou University (No. Lzu07002).

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